Trimetric Theory: Equilateral Tonal Symmetry in Quaternary Chromatic Space

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Abstract

This paper introduces Trimetric Theory, a systematic approach to generating a novel, nine-note scale through the strategic modification of major scales built upon a major-third-related triadic foundation. By constructing major scales from three notes separated by major thirds and removing their second scale degrees, a nine-note collection emerges with distinct symmetrical properties. We present formal definitions, analyze the mathematical characteristics of the resulting scale system, demonstrate its cyclical modal properties, and outline its compositional applications. Empirical validation through dedicated compositions confirms the theory's musical coherence and practical utility.

1. Introduction

Traditional Western music theory primarily operates within seven-note diatonic systems. This paper introduces Trimetric Theory, which systematically generates a symmetrical nine-note scale that maintains internal coherence while expanding harmonic possibilities. The theory derives its name from the three-fold symmetrical properties that characterize its structural organization.

The Trimetric Scale constitutes a nine-note structure containing both a complete whole tone scale and elements of three major scales. This scale exhibits distinct properties including three-fold symmetry, complementary relationships with other scales in the system, and consistent internal structure across all possible formations. The construction method, based on modified major scales built on augmented triad foundations, produces a scale system with precise geometrical relationships.

Through formal theoretical analysis and compositional application, this paper defines the properties of the Trimetric Scale and demonstrates its applications within a systematic framework for expanded tonality.

2. Theoretical Framework

2.1 Definitions and Notation

The term "Trimetric" derives from the three-fold symmetrical properties that characterize this theoretical system. Unlike simple bilateral symmetry, Trimetric Theory exhibits equilateral or trigonometric symmetry across multiple dimensions: in its foundational three-note structure, in its three-fold modal repetition pattern, and in the systematic relationships between complementary scales. This multi-level trimetrical organization represents the fundamental mathematical property from which the theory's name originates.

1. **Trimetric Triad**: *Three notes separated by major thirds (4 semitones each): {N, N+4, N+8}. Occasionally denoted by the delta symbol (Δ) followed by one or all three notes of the triad.*
2. **Trimetric Scale (TS or Δ)**: *A nine-note scale derived from a Trimetric Triad. Denoted by the delta symbol (Δ) followed by the three notes of the triad, beginning with the initiating note N. Occasionally simplified by including only the first, or the reference, note of the triad.*
    - *Examples: ΔF♯A♯D, ΔA♯DF♯, ΔDF♯A♯ (rotational variants of the same scale)*
    - *Simplified Example: ΔF♯, ΔA♯, ΔD*
    - *There are exactly four unique Trimetric Scales in the twelve-tone system, each capable of three rotational positions*
3. **Modified Major Scale (MS-2)**: *A major scale with its second scale degree removed, resulting in a six-note collection.*
    - *Notation: C MS-2 represents C major scale minus its second degree (D)*
    - *Example: C MS-2 = {C, E, F, G, A, B}*
4. **Trimetric Mode**: *Any of the nine rotational permutations of the Trimetric Scale.*

2.2 The Trimetric Transform Operation

Definition 2.1 (Trimetric Transform): *For any note N, the Trimetric Transform T(N) is defined as follows:*

1. *Generate the Trimetric Triad: {N, N+4, N+8} (where +4 and +8 represent semitone intervals)*
2. *For each X in the triad, construct the major scale MS(X)*
3. *For each MS(X), remove the 2nd scale degree to produce the modified scale MS-2(X)*
4. *The Trimetric Scale is the union of these modified scales: $T(N) = \cup \{MS\text{-}2(X) \mid X \in triad\}$*

This can be expressed algorithmically as:

Algorithm 1: Trimetric Transform

Input: Note N
Output: Nine-note Trimetric Scale ΔN

```
1. Triad ← {N, N+4, N+8}
2. Initialize empty set ΔN
3. For each X in Triad:
   a. MS(X) ← MajorScale(X)
   b. MS-2(X) ← MS(X) - {2nd degree of MS(X)}
         // Generate major scales minus the second interval degree
   c. ΔN ← ΔN ∪ MS-2(X)
         // Unite the Triad with the major scales
4. Return ΔN
```

2.3 Alternative Construction Method

The Trimetric Scale can also be constructed through an alternative but equivalent method:

Definition 2.2 (Alternative Construction): *For any note N, the Trimetric Scale ΔN can be constructed thus:*

1. *Generate the Trimetric Triad: {N, N+4, N+8}*
2. *Identify the whole tone scale starting one semitone above N: WT(N+1)*
3. *The Trimetric Scale is the union: ΔN = Triad ∪ WT(N+1)*

Theorem 1 (Construction Equivalence): *The two construction methods produce identical results.*

Proof:

1. For any note N, its MS-2 contains a subset of notes from one whole tone scale plus notes from the other whole tone scale.
2. When three MS-2 scales from notes separated by major thirds are combined, they collectively contain one complete whole tone scale plus the three triad notes.
3. This equals precisely the set produced by the alternative construction method.
4. Therefore, the construction methods are equivalent. ■

Algorithm 2: Alternative Trimetric Construction

Input: Note N
Output: Nine-note Trimetric Scale ΔN

1. Triad ← {N, N+4, N+8}
2. Initialize empty set ΔN
3. WT(N+1) ← Whole Tone Scale (N+1)
   // Generate whole tone scale starting one semitone above N
4. ΔN ← Triad ∪ WT(N+1)
   // Unite the triad with the whole tone scale
5. Return ΔN

2.4 Mathematical Properties

Theorem 2 (Structural Composition): *Every Trimetric Scale consists of:*

1. *A complete whole tone scale (6 notes)*
2. *Three notes from a major-third-related Trimetric Triad*

Proof:

1. When three major scales with their seconds removed are combined, exactly 9 unique notes result.

2. These 9 notes can be decomposed into a complete whole tone scale (6 notes) plus the 3 notes of the Trimetric Triad.
3. This structure is invariant across all possible Trimetric Scales.
4. The 3 removed notes from the chromatic scale always form another Trimetric Triad. ■

Theorem 3 (Cardinality Constraint): *There exist exactly four unique Trimetric Scales, each capable of three rotational positions.*

Proof:

1. Each Trimetric Scale is defined by a Trimetric Triad consisting of notes separated by major thirds.
2. In the 12-tone chromatic system, there are exactly four unique Trimetric Triads:
    - ΔCEG♯
    - ΔC♯FA
    - ΔDF♯A♯
    - ΔD♯GB
3. Each triad can appear in three rotational positions without changing the resulting scale structure.
4. Therefore, there are exactly four unique Trimetric Scales, each with three rotational variants. ■

Theorem 4 (Modal Cyclicity): *The interval structure of Trimetric modes repeats every three modes, creating exactly three distinct modal families.*

Proof:

1. For any Trimetric Scale ΔXYZ, the modes starting on notes X, Y, and Z have unique interval structures.
2. The mode starting on the note a semitone above X shares the same interval structure as the mode starting on the note a semitone above Y.
3. The mode starting on the note a semitone above Y shares the same interval structure as the mode starting on the note a semitone above Z.
4. The mode starting on the note a semitone above Z shares the same interval structure as the mode starting on the note a semitone above X.
5. For a note in ΔXYZ other than semitones above X, Y, and Z, this pattern also applies. In cases where there are no semitones above X, Y, or Z, the pattern applies to the nearest whole tone above.
6. This pattern continues through all nine notes of the scale.
7. Therefore, despite having nine notes, the Trimetric Scale contains only three distinct modal structures that repeat in cycle.
8. Each of these three modal families retains characteristics of traditional diatonic modes with specific alterations due to the MS-2 construction method. ■

Theorem 5 (Complementary Symmetry): *The four unique Trimetric Scales form two complementary pairs with perfect symmetrical properties.*

Proof:

1. For any Trimetric Scale ΔN based on triad {N, N+4, N+8}, there exists a complementary Trimetric Scale ΔM based on triad {M, M+4, M+8}.
2. The notes of triad {M, M+4, M+8} are precisely the three notes absent from ΔN.
3. Similarly, the notes of triad {N, N+4, N+8} are the three notes absent from ΔM.
4. Both ΔN and ΔM share the same six-note whole tone scale.
5. Therefore, the 12-tone chromatic system is perfectly partitioned by each complementary pair: two Trimetric Triads (6 notes) and their shared whole tone scale (6 notes). ■

3. Analytical Examples

3.1 C-Based Trimetric Scale

Let us apply the Trimetric Transform to the note C:

1. Triad = {C, E, G♯}
2. Modified Major Scales:
   - C MS-2 = {C, E, F, G, A, B}
   - E MS-2 = {E, G♯, A, B, C♯, D♯}
   - G♯ MS-2 = {G♯, C, C♯, D♯, F, G}
3. Trimetric Scale: ΔC = {C, C♯, D♯, E, F, G, G♯, A, B}

Using the alternative construction method:

1. Triad = {C, E, G♯}
2. Whole tone scale starting on C♯: {C♯, D♯, F, G, A, B}
3. ΔC = {C, E, G♯} ∪ {C♯, D♯, F, G, A, B} = {C, C♯, D♯, E, F, G, G♯, A, B}

Both methods yield the identical nine-note Trimetric Scale.

3.2 Visual Representation

The formation of a Trimetric Scale can be visualized by mapping the three component modified major scales onto the chromatic spectrum:

Table 1: Formation of ΔCEG♯ Trimetric Scale

	C	**C♯**	**D**	**D♯**	**E**	**F**	**F♯**	**G**	**G♯**	**A**	**A♯**	**B**
C MS-2	**C**				**E**	**F**		**G**		**A**		**B**
E MS-2		**C♯**		**D♯**	**E**				**G♯**	**A**		**B**
G♯ MS-2	**C**	**C♯**		**D♯**		**F**		**G**	**G♯**			

Trimetric Scale	**C**	**C♯**		**D♯**	**E**	**F**		**G**	**G♯**	**A**		**B**

This representation demonstrates how each modified major scale contributes to the complete Trimetric Scale and reveals the systematic coverage of 9 out of 12 chromatic pitches.

Visualization of the Trimetric Scale reveals additional patterns characteristic of its unique structure. Each note in the Trimetric Scale is contained within exactly two of the three instances of MS-2. Additionally, the Trimetric scale can be depicted broadly as a series of three groups of three chromatically ascending notes, where each group is separated by a whole tone.

3.3 Complementary Trimetric Scales

The Trimetric Scale ΔCEG♯ lacks the notes D, F♯, and A♯—precisely the notes that form another Trimetric Triad {D, F♯, A♯}. This complementary relationship extends to all Trimetric Scales, creating two perfect pairs:

Table 2: Relationship between ΔCEG♯ and ΔDF♯A♯

Note	C	C♯	D	D♯	E	F	F♯	G	G♯	A	A♯	B
ΔCEG♯	**C**	C♯		D♯	**E**	F		G	**G♯**	A		B
ΔDF♯A♯		C♯	**D**	D♯		F	**F♯**	G		A	**A♯**	B
Whole Tone 2		C♯		D♯		F		G		A		B
ΔCEG♯ Triad Only	**C**				**E**				**G♯**			
ΔDF♯A♯ Triad Only			**D**				**F♯**				**A♯**	

Table 3: Relationship between ΔC♯FA and ΔD♯GB

Note	C	C♯	D	D♯	E	F	F♯	G	G♯	A	A♯/B♭	B
ΔC♯FA	C	**C♯**	D		E	**F**	F♯		G♯	**A**		B
ΔD♯GB			D	**D♯**	E		F♯	**G**			B♭	**B**
Whole Tone 1	C		D		E		F♯		G♯		A♯	
ΔC♯FA Triad Only		**C♯**				**F**				**A**		

ΔD♯GB Triad Only				**D♯**				**G**				**B**

Each complementary pair shares one of the two possible whole tone scales, creating a symmetrical relationship: each Trimetric Scale consists of its defining triad (3 notes) plus a complete whole tone scale (6 notes). Meanwhile, its complementary scale contains the exact notes absent from the first triad's scale, plus the same whole tone scale.

Each complementary pair of Trimetric Triads, when combined, produce the whole tone scale which is complementary to that contained in both of the complementary scales.

4. Modal Properties

4.1 Three-Fold Modal Symmetry

The Trimetric Scale exhibits a three-fold symmetry in its modal structure:

1. Despite having nine notes, the nine modes group into three distinct families
2. Modes built on notes separated by major thirds share identical interval structures
3. This creates a cyclical pattern that repeats every three positions in the scale

The traditional modal framework (Ionian, Dorian, etc.) applies only partially to Trimetric modes. Each mode contains altered scale degrees and multiple options for certain positions. These alterations and additional options create a harmonic environment where traditional modal functions are partially preserved while simultaneously expanded.

4.2 Integrated Trimetric Grids

As demonstrated in the Key Squared Generator output, Trimetric Scales can be represented in grid form:

Trimetric Scale: G♯ A B C C♯ D♯ E F G

```
This Trimetric scale belongs to the ΔCEG# group.

Integrated Trimetric Scale: E F G G# A B C C# D#

Integrated Trimetric Scale Grid (ΔEG#C)^2:

     |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
  E  |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#
|
  F  |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E
|
```

```
  G  |   G    |  G#    |  A#    |   B    |   C    |   D    |  D#    |   E    |  F#
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
 G#  |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |   G
|
  A  |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#
|
  B  |   B    |   C    |   D    |  D#    |   E    |  F#    |   G    |  G#    |  A#
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
  C  |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B
|
 C#  |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C
|
 D#  |  D#    |   E    |  F#    |   G    |  G#    |  A#    |   B    |   C    |   D
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
```

Integrated Trimetric Mode Grid ΔEG#C (Modal):

```
     |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
  E  |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#
|
  F  |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E
|
  G  |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |   F
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
 G#  |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |   G
|
  A  |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#
|
  B  |   B    |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
  C  |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B
|
 C#  |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |   C
|
 D#  |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#
|
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------
++
```

The grid format reveals both the complete Trimetric system and its internal symmetrical relationships, facilitating analysis of modal relationships across the nine tonal centers.

5. Compositional Applications

5.1 Harmonic Properties and Applications

The Trimetric Scale offers specific harmonic capabilities stemming from its unique structure:

1. **Triad-Scale Connectivity**: Each Trimetric Scale functions as a bridge between three equidistant MS-2 scales, allowing direct modulation between these otherwise distant tonal centers.
2. **Whole Tone Integration**: The embedded whole tone scale creates systematic connectivity between complementary Trimetric Scales, enabling movement between paired tonal systems.
3. **Chromatic Relationships**: Chromatically-separated Trimetric Scales offer additional systematic relationships which, when combined with their own parallel inverted triad-scale connectivity and whole tone integration, as well as their shared scale structures among adjacent Trimetric Scales, provide a comprehensive framework across all Trimetric Scales for both harmonic and melodic analysis.

5.2 Case Study: Trimetry Album

A complete concept album has been composed using Trimetric Theory (Simpson, 2023), demonstrating the practical application of this system for use in composition.

The compositions methodically explore diverse aspects of Trimetric Theory through various constraints and structural approaches, including: exclusive use of MS-2 scales; modal variations; explicit employment of the embedded whole tone structure; and systematic investigation of relationships between adjacent or complementary Trimetric Scales. This comprehensive exploration tests the theoretical principles across multiple compositional contexts, confirming their versatility and coherence.

The album employs an allegorical framework wherein Trimetric scales and their symmetry properties are anthropomorphized within the context of a storyline to demonstrate their relationships.

Analysis confirms the musical coherence and practical utility of the system.

6. Relationship to Integration Theory

Trimetric Theory relates to Integration Theory (Simpson, 2024) in several ways:

1. Both theories expand traditional harmony through systematic transformational operations
2. Both utilize the major third interval as a key component of their transformational operations
3. While Integration Theory transforms one-dimensional modal options into a two-dimensional space, Trimetric Theory generates a symmetrical nine-note scale composed of three partial major scales

4. The resulting Trimetric Scale exhibits symmetrical properties not found in conventional diatonic or octatonic systems
5. The Key Squared Generator application incorporates Trimetric Theory through its "Secret Key" feature

These connections suggest a broader framework for analytical approaches to expanded tonal systems.

7. Software Implementation

The "Secret Key" feature in the Key Squared Generator (Simpson, 2024) implements Trimetric Theory, providing:

1. Automatic generation of Trimetric Scales from any starting note
2. Grid representation of modal relationships
3. Analysis tools for compositional application
4. Visual representation of complementary scale relationships

7.1 Example: C Trimetric Scale

The Key Squared Generator was used to generate the following example. The response displays the “Secret Key” information that is revealed when entering a Trimetric Scale as Key input.

```
Enter Key (optional): c c# d# e f g g# a b

Welcome to the Land of Trimetry!

The Trimetric Scale is symmetrical with 9 notes.
Example: C C# D# E F G G# A B.

Embedded in each Trimetric Scale are three distinct Major Scales - without their 2nd
notes.
Example: 'C _ E F G A B' || 'E _ G# A B C# D#' || 'G# _ C C# D# F G'

This Trimetric scale belongs to the ΔCEG# group.

Integrated Trimetric Scale: G# A B C C# D# E F G

Integrated Trimetric Scale Grid (ΔG#CE)^2:

     |  G#  |  A   |  B   |  C   |  C#  |  D#  |  E   |  F   |  G   |
-----+------+------+------+------+------+------+------+------+------+
 G#  |  G#  |  A   |  B   |  C   |  C#  |  D#  |  E   |  F   |  G   |
 A   |  A   |  A#  |  C   |  C#  |  D   |  E   |  F   |  F#  |  G#  |
 B   |  B   |  C   |  D   |  D#  |  E   |  F#  |  G   |  G#  |  A#  |
-----+------+------+------+------+------+------+------+------+------+
 C   |  C   |  C#  |  D#  |  E   |  F   |  G   |  G#  |  A   |  B   |
 C#  |  C#  |  D   |  E   |  F   |  F#  |  G#  |  A   |  A#  |  C   |
 D#  |  D#  |  E   |  F#  |  G   |  G#  |  A#  |  B   |  C   |  D   |
-----+------+------+------+------+------+------+------+------+------+
 E   |  E   |  F   |  G   |  G#  |  A   |  B   |  C   |  C#  |  D#  |
```

```
 F   |  F   |  F#  |  G#  |  A   |  A#  |  C   |  C#  |  D   |  E   |
 G   |  G   |  G#  |  A#  |  B   |  C   |  D   |  D#  |  E   |  F#  |
-----+------+------+------+------+------+------+------+------+------+

Integrated Trimetric Mode Grid ΔG#CE (Modal):

     |  G#  |  A   |  B   |  C   |  C#  |  D#  |  E   |  F   |  G   |
-----+------+------+------+------+------+------+------+------+------+
 G#  |  G#  |  A   |  B   |  C   |  C#  |  D#  |  E   |  F   |  G   |
 A   |  A   |  B   |  C   |  C#  |  D#  |  E   |  F   |  G   |  G#  |
 B   |  B   |  C   |  C#  |  D#  |  E   |  F   |  G   |  G#  |  A   |
-----+------+------+------+------+------+------+------+------+------+
 C   |  C   |  C#  |  D#  |  E   |  F   |  G   |  G#  |  A   |  B   |
 C#  |  C#  |  D#  |  E   |  F   |  G   |  G#  |  A   |  B   |  C   |
 D#  |  D#  |  E   |  F   |  G   |  G#  |  A   |  B   |  C   |  C#  |
-----+------+------+------+------+------+------+------+------+------+
 E   |  E   |  F   |  G   |  G#  |  A   |  B   |  C   |  C#  |  D#  |
 F   |  F   |  G   |  G#  |  A   |  B   |  C   |  C#  |  D#  |  E   |
 G   |  G   |  G#  |  A   |  B   |  C   |  C#  |  D#  |  E   |  F   |
-----+------+------+------+------+------+------+------+------+------+
```

This example demonstrates output generated for the specified Trimetric Scale. Both tables illustrate multiple areas of symmetry and redundancy, where the second table is a subset of the first:

1. The first table displays Integrated data (Simpson, 2025) from input parameters, specifically designating intervallic notes of the Integrated Trimetric scale as tonal centers of their own Trimetric Scales, in a similar fashion to the Integration Operation.
2. The second table displays Integrated data from input parameters, specifically organizing the modes within the Integrated Trimetric Scale.

8. Future Research Directions

The emergent symmetrical properties of Trimetric Theory suggest several promising avenues for further investigation:

1. **Dimensional Transformations**: The relationship between note-level, scale-level, and higher-dimensional transformations may reveal further underlying structural principles connecting Trimetric Theory with other systematic approaches to harmony.
2. **Inter-Scale Relationships**: Beyond complementary pairs, the relationships between adjacent Trimetric Scales (e.g., ΔC and ΔC♯) warrant further exploration for their compositional implications.
3. **Modal Classification Systems**: The transformed nature of traditional modes within the Trimetric framework suggests the potential for developing a more comprehensive modal taxonomy specific to this nine-note symmetrical system.
4. **Unified Theoretical Framework**: The connections between Trimetric Theory and Integration Theory point toward the possibility of a unified field theory of tonal systems, encompassing multiple approaches to systematic harmony. A potentially valuable application would be computational tools that employ Integration Theory for diatonic analysis and extend to Trimetric Theory when harmonic requirements exceed diatonic constraints. This could allow for

counterpoint analysis that considers both melodic and bass lines simultaneously, providing systematic chromatic solutions when diatonic options are exhausted.

These research directions could further extend the theoretical foundation established in this paper while expanding the practical applications of Trimetric Theory in composition and analysis.

9. Conclusion

Trimetric Theory provides a systematic approach to generating and applying a nine-note scale system with distinct mathematical properties. The resulting Trimetric Scales offer expanded harmonic resources while maintaining internal coherence through their three-fold symmetry, embedded major scale structure, and complementary relationships.

This theory represents an extension to traditional diatonic systems, offering composers a formally rigorous framework for exploring expanded tonality. The symmetry between complementary Trimetric Scales and their shared whole tone components reveals an elegant organizational structure underlying the system.

Empirical validation through composed works confirms that Trimetric Theory provides both theoretical coherence and practical compositional utility.

References

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Appendix A: Complete Catalog of Trimetric Scales and Modal Families

This section contains tables of all Integrated Trimetric Scales with their complete note sets.

This section also includes analysis of all modal families with their characteristic intervals and harmonic tendencies.

1. C C# D# E F G G# A B

This Trimetric scale belongs to the ΔCEG# group.

Integrated Trimetric Scale: G# A B C C# D# E F G

Integrated Trimetric Scale Grid (ΔG#CE)^2:

	G#	A	B	C	C#	D#	E	F	G
G#	G#	A	B	C	C#	D#	E	F	G
A	A	A#	C	C#	D	E	F	F#	G#
B	B	C	D	D#	E	F#	G	G#	A#
C	C	C#	D#	E	F	G	G#	A	B
C#	C#	D	E	F	F#	G#	A	A#	C
D#	D#	E	F#	G	G#	A#	B	C	D
E	E	F	G	G#	A	B	C	C#	D#
F	F	F#	G#	A	A#	C	C#	D	E
G	G	G#	A#	B	C	D	D#	E	F#

Integrated Trimetric Mode Grid ΔG#CE (Modal):

	G#	A	B	C	C#	D#	E	F	G
G#	G#	A	B	C	C#	D#	E	F	G
A	A	B	C	C#	D#	E	F	G	G#
B	B	C	C#	D#	E	F	G	G#	A
C	C	C#	D#	E	F	G	G#	A	B
C#	C#	D#	E	F	G	G#	A	B	C

D#	D#	E	F	G	G#	A	B	C	C#
E	E	F	G	G#	A	B	C	C#	D#
F	F	G	G#	A	B	C	C#	D#	E
G	G	G#	A	B	C	C#	D#	E	F

2. C# D E F F# G# A A# C

This Trimetric scale belongs to the ΔC#FA group.

Integrated Trimetric Scale: A A# C C# D E F F# G#

Integrated Trimetric Scale Grid (ΔAC#F)^2:

	A	A#	C	C#	D	E	F	F#	G#
A	A	A#	C	C#	D	E	F	F#	G#
A#	A#	B	C#	D	D#	F	F#	G	A
C	C	C#	D#	E	F	G	G#	A	B
C#	C#	D	E	F	F#	G#	A	A#	C
D	D	D#	F	F#	G	A	A#	B	C#
E	E	F	G	G#	A	B	C	C#	D#
F	F	F#	G#	A	A#	C	C#	D	E
F#	F#	G	A	A#	B	C#	D	D#	F
G#	G#	A	B	C	C#	D#	E	F	G

Integrated Trimetric Mode Grid ΔAC#F (Modal):

	A	A#	C	C#	D	E	F	F#	G#

```
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  A  |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |
 A#  |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |
  C  |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 C#  |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |
  D  |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |
  E  |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  F  |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
 F#  |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |
 G#  |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

3. D D# F F# G A A# B C#

This Trimetric scale belongs to the ΔDF#A# group.

Integrated Trimetric Scale: A# B C# D D# F F# G A

Integrated Trimetric Scale Grid (ΔA#DF#)^2:

```
     |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 A#  |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |
  B  |   B    |   C    |   D    |  D#    |   E    |  F#    |   G    |  G#    |  A#    |
 C#  |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  D  |   D    |  D#    |   F    |  F#    |   G    |   A    |  A#    |   B    |  C#    |
 D#  |  D#    |   E    |  F#    |   G    |  G#    |  A#    |   B    |   C    |   D    |
  F  |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 F#  |  F#    |   G    |   A    |  A#    |   B    |  C#    |   D    |  D#    |   F    |
```

```
  G  |   G    |  G#    |  A#    |   B    |   C    |   D    |  D#    |   E    |  F#    |
  A  |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

Integrated Trimetric Mode Grid ΔA#DF# (Modal):

```
     |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 A#  |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |
  B  |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |  A#    |
 C#  |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |  A#    |   B    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  D  |   D    |  D#    |   F    |  F#    |   G    |   A    |  A#    |   B    |  C#    |
 D#  |  D#    |   F    |  F#    |   G    |   A    |  A#    |   B    |  C#    |   D    |
  F  |   F    |  F#    |   G    |   A    |  A#    |   B    |  C#    |   D    |  D#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 F#  |  F#    |   G    |   A    |  A#    |   B    |  C#    |   D    |  D#    |   F    |
  G  |   G    |   A    |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |
  A  |   A    |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

4. D# E F# G G# A# B C D

This Trimetric scale belongs to the ΔD#GB group.

Integrated Trimetric Scale: B C D D# E F# G G# A#

Integrated Trimetric Scale Grid (ΔBD#G)^2:

```
     |   B    |   C    |   D    |  D#    |   E    |  F#    |   G    |  G#    |  A#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  B  |   B    |   C    |   D    |  D#    |   E    |  F#    |   G    |  G#    |  A#    |
  C  |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |
```

D	D	D#	F	F#	G	A	A#	B	C#	
D#	D#	E	F#	G	G#	A#	B	C	D	
E	E	F	G	G#	A	B	C	C#	D#	
F#	F#	G	A	A#	B	C#	D	D#	F	
G	G	G#	A#	B	C	D	D#	E	F#	
G#	G#	A	B	C	C#	D#	E	F	G	
A#	A#	B	C#	D	D#	F	F#	G	A	

Integrated Trimetric Mode Grid ∆BD#G (Modal):

	B	C	D	D#	E	F#	G	G#	A#
B	B	C	D	D#	E	F#	G	G#	A#
C	C	D	D#	E	F#	G	G#	A#	B
D	D	D#	E	F#	G	G#	A#	B	C
D#	D#	E	F#	G	G#	A#	B	C	D
E	E	F#	G	G#	A#	B	C	D	D#
F#	F#	G	G#	A#	B	C	D	D#	E
G	G	G#	A#	B	C	D	D#	E	F#
G#	G#	A#	B	C	D	D#	E	F#	G
A#	A#	B	C	D	D#	E	F#	G	G#

5. E F G G# A B C C# D#

This Trimetric scale belongs to the ∆CEG# group.

Integrated Trimetric Scale: C C# D# E F G G# A B

Integrated Trimetric Scale Grid (ΔCEG#)^2:

	C	C#	D#	E	F	G	G#	A	B
C	C	C#	D#	E	F	G	G#	A	B
C#	C#	D	E	F	F#	G#	A	A#	C
D#	D#	E	F#	G	G#	A#	B	C	D
E	E	F	G	G#	A	B	C	C#	D#
F	F	F#	G#	A	A#	C	C#	D	E
G	G	G#	A#	B	C	D	D#	E	F#
G#	G#	A	B	C	C#	D#	E	F	G
A	A	A#	C	C#	D	E	F	F#	G#
B	B	C	D	D#	E	F#	G	G#	A#

Integrated Trimetric Mode Grid ΔCEG# (Modal):

	C	C#	D#	E	F	G	G#	A	B
C	C	C#	D#	E	F	G	G#	A	B
C#	C#	D#	E	F	G	G#	A	B	C
D#	D#	E	F	G	G#	A	B	C	C#
E	E	F	G	G#	A	B	C	C#	D#
F	F	G	G#	A	B	C	C#	D#	E
G	G	G#	A	B	C	C#	D#	E	F

```
 G#  |  G#     |  A      |  B      |  C      |  C#     |  D#     |  E      |  F      |  G      |

  A  |   A     |  B      |  C      |  C#     |  D#     |  E      |  F      |  G      |  G#     |

  B  |   B     |  C      |  C#     |  D#     |  E      |  F      |  G      |  G#     |  A      |

-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

6. F F# G# A A# C C# D E

This Trimetric scale belongs to the ΔC#FA group.

Integrated Trimetric Scale: C# D E F F# G# A A# C

Integrated Trimetric Scale Grid (ΔC#FA)^2:

```
     |  C#     |  D      |  E      |  F      |  F#     |  G#     |  A      |  A#     |  C      |

-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++

 C#  |  C#     |  D      |  E      |  F      |  F#     |  G#     |  A      |  A#     |  C      |

  D  |   D     |  D#     |  F      |  F#     |  G      |  A      |  A#     |  B      |  C#     |

  E  |   E     |  F      |  G      |  G#     |  A      |  B      |  C      |  C#     |  D#     |

-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++

  F  |   F     |  F#     |  G#     |  A      |  A#     |  C      |  C#     |  D      |  E      |

 F#  |  F#     |  G      |  A      |  A#     |  B      |  C#     |  D      |  D#     |  F      |

 G#  |  G#     |  A      |  B      |  C      |  C#     |  D#     |  E      |  F      |  G      |

-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++

  A  |   A     |  A#     |  C      |  C#     |  D      |  E      |  F      |  F#     |  G#     |

 A#  |  A#     |  B      |  C#     |  D      |  D#     |  F      |  F#     |  G      |  A      |

  C  |   C     |  C#     |  D#     |  E      |  F      |  G      |  G#     |  A      |  B      |

-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

Integrated Trimetric Mode Grid ΔC#FA (Modal):

```
     |  C#     |  D      |  E      |  F      |  F#     |  G#     |  A      |  A#     |  C      |

-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++

 C#  |  C#     |  D      |  E      |  F      |  F#     |  G#     |  A      |  A#     |  C      |
```

D	D	E	F	F#	G#	A	A#	C	C#	
E	E	F	F#	G#	A	A#	C	C#	D	
F	F	F#	G#	A	A#	C	C#	D	E	
F#	F#	G#	A	A#	C	C#	D	E	F	
G#	G#	A	A#	C	C#	D	E	F	F#	
A	A	A#	C	C#	D	E	F	F#	G#	
A#	A#	C	C#	D	E	F	F#	G#	A	
C	C	C#	D	E	F	F#	G#	A	A#	

7. F# G A A# B C# D D# F

This Trimetric scale belongs to the ΔDF#A# group.

Integrated Trimetric Scale: D D# F F# G A A# B C#

Integrated Trimetric Scale Grid (ΔDF#A#)^2:

	D	D#	F	F#	G	A	A#	B	C#	
D	D	D#	F	F#	G	A	A#	B	C#	
D#	D#	E	F#	G	G#	A#	B	C	D	
F	F	F#	G#	A	A#	C	C#	D	E	
F#	F#	G	A	A#	B	C#	D	D#	F	
G	G	G#	A#	B	C	D	D#	E	F#	
A	A	A#	C	C#	D	E	F	F#	G#	
A#	A#	B	C#	D	D#	F	F#	G	A	
B	B	C	D	D#	E	F#	G	G#	A#	
C#	C#	D	E	F	F#	G#	A	A#	C	

Integrated Trimetric Mode Grid ΔDF#A# (Modal):

	D	D#	F	F#	G	A	A#	B	C#
D	D	D#	F	F#	G	A	A#	B	C#
D#	D#	F	F#	G	A	A#	B	C#	D
F	F	F#	G	A	A#	B	C#	D	D#
F#	F#	G	A	A#	B	C#	D	D#	F
G	G	A	A#	B	C#	D	D#	F	F#
A	A	A#	B	C#	D	D#	F	F#	G
A#	A#	B	C#	D	D#	F	F#	G	A
B	B	C#	D	D#	F	F#	G	A	A#
C#	C#	D	D#	F	F#	G	A	A#	B

8. G G# A# B C D D# E F#

This Trimetric scale belongs to the ΔD#GB group.

Integrated Trimetric Scale: D# E F# G G# A# B C D

Integrated Trimetric Scale Grid (ΔD#GB)^2:

	D#	E	F#	G	G#	A#	B	C	D
D#	D#	E	F#	G	G#	A#	B	C	D
E	E	F	G	G#	A	B	C	C#	D#
F#	F#	G	A	A#	B	C#	D	D#	F

```
  G  |   G    | G#     | A#     |  B     |  C     |  D     | D#     |  E     | F#     |
 G#  |  G#    |  A     |  B     |  C     | C#     | D#     |  E     |  F     |  G     |
 A#  |  A#    |  B     | C#     |  D     | D#     |  F     | F#     |  G     |  A     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  B  |   B    |  C     |  D     | D#     |  E     | F#     |  G     | G#     | A#     |
  C  |   C    | C#     | D#     |  E     |  F     |  G     | G#     |  A     |  B     |
  D  |   D    | D#     |  F     | F#     |  G     |  A     | A#     |  B     | C#     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

Integrated Trimetric Mode Grid ΔD#GB (Modal):

```
     |  D#    |  E     | F#     |  G     | G#     | A#     |  B     |  C     |  D     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 D#  |  D#    |  E     | F#     |  G     | G#     | A#     |  B     |  C     |  D     |
  E  |   E    | F#     |  G     | G#     | A#     |  B     |  C     |  D     | D#     |
 F#  |  F#    |  G     | G#     | A#     |  B     |  C     |  D     | D#     |  E     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  G  |   G    | G#     | A#     |  B     |  C     |  D     | D#     |  E     | F#     |
 G#  |  G#    | A#     |  B     |  C     |  D     | D#     |  E     | F#     |  G     |
 A#  |  A#    |  B     |  C     |  D     | D#     |  E     | F#     |  G     | G#     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  B  |   B    |  C     |  D     | D#     |  E     | F#     |  G     | G#     | A#     |
  C  |   C    |  D     | D#     |  E     | F#     |  G     | G#     | A#     |  B     |
  D  |   D    | D#     |  E     | F#     |  G     | G#     | A#     |  B     |  C     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

9. G# A B C C# D# E F G

This Trimetric scale belongs to the ΔCEG# group.

Integrated Trimetric Scale: E F G G# A B C C# D#

Integrated Trimetric Scale Grid (ΔEG#C)^2:

```
     |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  E  |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |
  F  |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
  G  |   G    |  G#    |  A#    |   B    |   C    |   D    |  D#    |   E    |  F#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 G#  |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |   G    |
  A  |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |
  B  |   B    |   C    |   D    |  D#    |   E    |  F#    |   G    |  G#    |  A#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  C  |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |
 C#  |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |
 D#  |  D#    |   E    |  F#    |   G    |  G#    |  A#    |   B    |   C    |   D    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

Integrated Trimetric Mode Grid ΔEG#C (Modal):

```
     |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  E  |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |
  F  |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |
  G  |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 G#  |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |   G    |
  A  |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |
  B  |   B    |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  C  |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |
 C#  |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |
```

```
 D#  |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

10. A A# C C# D E F F# G#

This Trimetric scale belongs to the ΔC#FA group.

Integrated Trimetric Scale: F F# G# A A# C C# D E

Integrated Trimetric Scale Grid (ΔFAC#)^2:

```
     |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  F  |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
 F#  |  F#    |   G    |   A    |  A#    |   B    |  C#    |   D    |  D#    |   F    |
 G#  |  G#    |   A    |   B    |   C    |  C#    |  D#    |   E    |   F    |   G    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  A  |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |
 A#  |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |
  C  |   C    |  C#    |  D#    |   E    |   F    |   G    |  G#    |   A    |   B    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 C#  |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |
  D  |   D    |  D#    |   F    |  F#    |   G    |   A    |  A#    |   B    |  C#    |
  E  |   E    |   F    |   G    |  G#    |   A    |   B    |   C    |  C#    |  D#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

Integrated Trimetric Mode Grid ΔFAC# (Modal):

```
     |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  F  |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
 F#  |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |
 G#  |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |
```

```
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  A  |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |
 A#  |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |
  C  |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 C#  |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |
  D  |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |
  E  |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

11. A# B C# D D# F F# G A

```
This Trimetric scale belongs to the ΔDF#A# group.

Integrated Trimetric Scale: F# G A A# B C# D D# F

Integrated Trimetric Scale Grid (ΔF#A#D)^2:
```

```
     |  F#    |   G    |   A    |  A#    |   B    |  C#    |   D    |  D#    |   F    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 F#  |  F#    |   G    |   A    |  A#    |   B    |  C#    |   D    |  D#    |   F    |
  G  |   G    |  G#    |  A#    |   B    |   C    |   D    |  D#    |   E    |  F#    |
  A  |   A    |  A#    |   C    |  C#    |   D    |   E    |   F    |  F#    |  G#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 A#  |  A#    |   B    |  C#    |   D    |  D#    |   F    |  F#    |   G    |   A    |
  B  |   B    |   C    |   D    |  D#    |   E    |  F#    |   G    |  G#    |  A#    |
 C#  |  C#    |   D    |   E    |   F    |  F#    |  G#    |   A    |  A#    |   C    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  D  |   D    |  D#    |   F    |  F#    |   G    |   A    |  A#    |   B    |  C#    |
 D#  |  D#    |   E    |  F#    |   G    |  G#    |  A#    |   B    |   C    |   D    |
  F  |   F    |  F#    |  G#    |   A    |  A#    |   C    |  C#    |   D    |   E    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

Integrated Trimetric Mode Grid ΔF#A#D (Modal):

```
     |  F#    |  G     |  A     |  A#    |  B     |  C#    |  D     |  D#    |  F     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 F#  |  F#    |  G     |  A     |  A#    |  B     |  C#    |  D     |  D#    |  F     |
  G  |  G     |  A     |  A#    |  B     |  C#    |  D     |  D#    |  F     |  F#    |
  A  |  A     |  A#    |  B     |  C#    |  D     |  D#    |  F     |  F#    |  G     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
 A#  |  A#    |  B     |  C#    |  D     |  D#    |  F     |  F#    |  G     |  A     |
  B  |  B     |  C#    |  D     |  D#    |  F     |  F#    |  G     |  A     |  A#    |
 C#  |  C#    |  D     |  D#    |  F     |  F#    |  G     |  A     |  A#    |  B     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  D  |  D     |  D#    |  F     |  F#    |  G     |  A     |  A#    |  B     |  C#    |
 D#  |  D#    |  F     |  F#    |  G     |  A     |  A#    |  B     |  C#    |  D     |
  F  |  F     |  F#    |  G     |  A     |  A#    |  B     |  C#    |  D     |  D#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
```

12. B C D D# E F# G G# A#

This Trimetric scale belongs to the ΔD#GB group.

Integrated Trimetric Scale: G G# A# B C D D# E F#

Integrated Trimetric Scale Grid (ΔGBD#)^2:

```
     |  G     |  G#    |  A#    |  B     |  C     |  D     |  D#    |  E     |  F#    |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  G  |  G     |  G#    |  A#    |  B     |  C     |  D     |  D#    |  E     |  F#    |
 G#  |  G#    |  A     |  B     |  C     |  C#    |  D#    |  E     |  F     |  G     |
 A#  |  A#    |  B     |  C#    |  D     |  D#    |  F     |  F#    |  G     |  A     |
-----+--------+--------+--------+--------+--------+--------+--------+--------+--------++
  B  |  B     |  C     |  D     |  D#    |  E     |  F#    |  G     |  G#    |  A#    |
  C  |  C     |  C#    |  D#    |  E     |  F     |  G     |  G#    |  A     |  B     |
```

D	D	D#	F	F#	G	A	A#	B	C#	
D#	D#	E	F#	G	G#	A#	B	C	D	
E	E	F	G	G#	A	B	C	C#	D#	
F#	F#	G	A	A#	B	C#	D	D#	F	

Integrated Trimetric Mode Grid ΔGBD# (Modal):

	G	G#	A#	B	C	D	D#	E	F#	
G	G	G#	A#	B	C	D	D#	E	F#	
G#	G#	A#	B	C	D	D#	E	F#	G	
A#	A#	B	C	D	D#	E	F#	G	G#	
B	B	C	D	D#	E	F#	G	G#	A#	
C	C	D	D#	E	F#	G	G#	A#	B	
D	D	D#	E	F#	G	G#	A#	B	C	
D#	D#	E	F#	G	G#	A#	B	C	D	
E	E	F#	G	G#	A#	B	C	D	D#	
F#	F#	G	G#	A#	B	C	D	D#	E	